

Investigation of Bouncing Universe and Phantom Crossing in Modified Gravity Coupled with Weyl Tensor and its Reconstruction

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Abstract

In this study, FRW cosmology in modified gravity containing arbitrary function $f(R)$ is taken into consideration when our action are coupled with Weyl tensor. It is indicated that the bouncing solution emerges in the model while the equation of state (EoS) parameter crosses the phantom divider. In this research, cosmological usage of the most promising candidates of dark energy in the framework of $f(R)$ theory coupled by Weyl tensor is explored. A $f(R)$ gravity model in the spatially flat FRW universe according to the ordinary version of the holographic dark energy model, which describes accelerated expansion of the universe is reconstructed. The equation of state parameter of the corresponding Weyl gravity models are obtained as well. We conclude that the holographic and Weyl gravity models can behave like phantom or quintessence models, whereas the equation of state parameter of the models can transit from quintessence state to phantom regime as shown recent observations.

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1. INTRODUCTION

One explanation for physicists with respect to how our universe formed is the big bang theory, commencing with Albert Einstein's theory of general relativity and bouncing theory in an alternative way of looking at how the universe commenced. The big bang theory tries to enunciate what the universe looked like before the planets and the stars came to existence. The universe, about 13.7 billion years ago, is believed by scientists to have condensed into a small region of matter and energy called a singularity. All of a sudden, the singularity exploded, expanding at an incredibly fast rate. Astronomers are of the belief that following the big bang, the universe was like a soup with small slices of matter floating around. The matter combined to create protogalaxies, which in turn combined to form galaxies. Within those galaxies, gas and dust blended to make stars. And around those stars, gravity pulled pieces of matter together to form planets.

The theory is simply the one which can not be recreated or proved by scientists at this point in time. Moreover, some detractors refer to a few weaknesses in big bang thinking. This signifies that other theories have been suggested for its replacement. Cosmologists are still attempting to predict the fate of our universe, trying to come to this conclusion whether it will expand forever, stabilize or collapse in on itself. Some cosmologists believe that the universe will eventually grow no more. Our universe will collapse in on itself into a singularity as gravity pulls matter down, an event billions of years from now which is called the big crunch. The planets, stars and galaxies in and of themselves are not dense enough to cause the big crunch. However, cosmologists believe that unseen materials exist and may exert enough gravitational force to stop the universe's expansion and cause the big crunch. The bouncing theory combines the big bang and big crunch theories to develop a vision of an infinite, cyclical cosmos in which the universe over and over again expands from a singularity only to ultimately collapse back in on itself, before doing it all over again. To put it another way, a bouncing universe would continuously expand and contract.

It should be mentioned that there is no evidence as to what will occur to our universe in the future, but about its beginning, it's different comparatively. Observational data of type Ia Super-Novae (SNIa) [1] have determined basic cosmological parameters in high-precisions. They are indicative of the fact that, the universe is spatially flat and dominated by two dark components containing dark energy and dark matter, and comprises nearly 73% dark

energy (DE) and 27% dust matter (cold dark matters plus baryons) with negligible radiations. Simultaneously, as to the origin of DE, they posed a fundamental problem. The combined analysis of SNIa [2], contingent upon the background expansion history of the universe around the redshift $z < 1$ as galaxy clusters measurements and Wilkinson Microwave Anisotropy Probe (WMAP) data [3], Sloan Digital Sky Survey (SDSS) [4], Chandra X-ray Observatory (CXO) [5] etc., it shows some cross-checked information of our universe, providing surprising proof as to the fact that the expansion of the universe for the time begin seems to have accelerating, behavior, being imputed to dark energy (DE), a strange energy with negative pressure. In contrast, dark matter (DM), a matter without pressure, is basically utilized to describe galactic curves and large-scale structure formation[6].

It is shown by the cosmological acceleration the present day universe is dominated by smoothly distributed slowly varying DE component. The constraint derived from SNIa has a degeneracy in the equation of state (EoS) of DE [7]. However, the nature of DE until now continues to be unknown, people have suggested some candidates for its explanation. The cosmological constant, Λ , in a model in which the universe's equation has a cosmological constant, indicated by Λ , and Cold Dark Matter (Λ CDM), is the most notable theoretical candidate of DE, having an equation of state with $\omega = -1$. This degeneracy is offered even by adding other constraints coming from Cosmic Microwave Background (CMB) [8] and Baryon Acoustic Oscillations (BAO) [9]. Astronomical observations denote that the cosmological constant, in their orders of magnitude, to be much smaller than it is calculated in modern theories of elementary particles [10]. Two of the most notable difficulties faced with the cosmological constant are the "fine-tuning" and the "cosmic coincidence" [11]. The constraints, nowadays, on the EoS around the cosmological constant value, $\omega = -1 \pm 0.1$ [6]-[12] and this probability exists that ω may differ in time. From the theoretical point of view there are three essentially different cases: $\omega > -1$ (quintessence), $\omega = -1$ (cosmological constant) and $\omega < -1$ (phantom) ([13]-[16] and refs. therein).

The models of DE can be generally categorized into two groups [17, 18]. In the first group, a specific matter leading to an accelerated expansion is introduced. Most of scalar field models such as quintessence [19] and k-essence [20] belong to this class. The second class, considered in this study, corresponds to the so-called modified gravity models such as $f(R)$ gravity [21], scalar-tensor theories [22] and brane-world models[23]. In order to break the degeneracy of observational constraints on ω and to discriminate between a DE models,

it is important to find additional information other than the background expansion history of the Universe [24].

Modified gravity, in the second classification [25], suggests fine alternative for DE origin. The expectation is that gravitational action has some extra terms which became relevant recently with the significant decrease of the universe curvature. The modified gravity can be obtained in three ways: first by substituting scalar curvature R , or by $f(R)$, second by taking additional curvature invariant terms into account like Gauss-Bonnet (GB) term as \mathcal{G} or $f(\mathcal{G})$ [26], and third by replacing a coupling of two methods ago as $f(R, \mathcal{G})$, in the Einstein-Hilbert action ($I_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R$) where G is gravitational constant.

In this regard, for a long time, conformal transformations and conformal techniques have been broadly in use in general relativity (Ref. [27] and references therein). It has often been claimed that conformal invariant field theories are renormalizable [28] and conformal gravity may be an alternative theory of gravity [29]. Because the gravitational field is long range and appears to travel with the speed of light, in the linear approximation, at least, it is expected for the equations to be conformally invariant. Einstein's theory of gravitation, known by all, is not conformally invariant. This theory appears not to be a totalizing universal theory of gravitational field because of the mentioned fact and some other issues emanating from standard cosmology and quantum field theory [30]. Many have tried to generalize this theory which goes back to the early days of general relativity (for reviewing see [31]). The first invariant gravitational theory under the scale transformation was presented by Weyl, being called Weyl gravity. The Conformal Weyl gravity is reliant upon local conformal invariance of the metric of the form $g_{\mu\nu}(x) \rightarrow \Omega^2(x)g_{\mu\nu}(x)$, being conducive to a theory with the field equation of fourth order derivative (higher-derivative theories) where Ω is independent of the space-time coordinates.

Some metric formulation of modified $f(R)$ gravities are suggested [25, 26], [32]-[35] describing the origin of cosmic acceleration. Particular attention is paid to $f(R)$ models [36]-[39] with the effective cosmological constant phase because such theories may easily reproduce the well-known Λ CDM cosmology. Such models subclass which does not violate Solar System tests represents the real alternative for standard General Relativity[40].

The Friedman equation, on the other hand, constitutes the starting point for nearly all researches in cosmology. The Friedman equation has been, corrected during the past few years being proposed in varying contexts, generally inspired by brane-world investigation

[41, 42]. These changes are often of a type that involves the total energy density ρ . In [43], multi-scalar coupled to gravity is studied in the context of conventional Friedman cosmology. It is found that the cosmological trajectories can be viewed as geodesic motion in an increased target space.

There are several phenomenological models which describe the crossing of the cosmological constant barrier [44, 45]. Finding a model following from the basic principles is of importance and which describes a crossing of the $\omega = -1$ barrier.

In this paper, in section 2, the dynamics of the FRW cosmology in modified gravity is considered. We discuss analytically and numerically a detailed examination of the conditions for having ω across over -1 . The necessary conditions required for a successful bounce is discussed in this section as well. In section 3 we will reconstruct our model corresponding to the Holographic Dark Energy (HDE) respectively. Finally, we summaries our paper in section 4.

2. THE MODEL

In the $f(R)$ theory of gravity the Einstein-Hilbert action is replaced by the square of the conformal Weyl tensor

$$I_W = -\frac{\alpha}{4} \int d^4x \sqrt{-g} \{C_{\mu\nu\rho\lambda} C^{\mu\nu\rho\lambda} + 2f(R)\}, \quad (1)$$

where $\alpha = 1/8\pi G$ and $C_{\mu\nu\rho\lambda}$ is the Weyl tensor

$$C_{\mu\nu\rho\lambda} = R_{\mu\nu\lambda\rho} - \frac{1}{2}(g_{\mu\lambda}R_{\nu\rho} - g_{\mu\rho}R_{\nu\lambda} - g_{\nu\lambda}R_{\mu\rho} + g_{\nu\rho}R_{\mu\lambda}) + \frac{R}{6}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}). \quad (2)$$

The action (1) can be written as flows

$$I_W = -\frac{\alpha}{4} \int d^4x \sqrt{-g} \left\{ R^{\mu\nu\rho\lambda} R_{\mu\nu\rho\lambda} - 2R^{\mu\nu} R_{\mu\nu} + \frac{1}{3}R^2 + 2f(R) \right\}, \quad (3)$$

since $\sqrt{-g}(R^{\mu\nu\rho\lambda} R_{\mu\nu\rho\lambda} - 4R^{\mu\nu} R_{\mu\nu} + R^2)$ is a total divergence (Gauss-Bonnet term), it does not contribute to the equation of motion and one can simplify the action as follows

$$\begin{aligned} I_W &= -\frac{\alpha}{2} \int d^4x \sqrt{-g} \left\{ R_{\mu\nu} R^{\mu\nu} - \frac{1}{3}R^2 + f(R) \right\} \\ &\equiv -\frac{\alpha}{2} \int d^4x \left\{ \mathcal{W}^{(2)} - \frac{1}{3}\mathcal{W}^{(1)} + \sqrt{-g}f(R) \right\}. \end{aligned} \quad (4)$$

The total action is $I \equiv I_W + I_M$, where I_M is the conformal matter action. Functional variation of the total action with regard to the matter fields produces the equations of motion while its functional variation considering the metric generates the $f(R)$ modified gravity coupled by Weyl field equation. Therefore, taking the variation of the action (4) with respect to the metric $g^{\mu\nu}$, the field equations can be obtained as [46],[47]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{\alpha}T_{\mu\nu}^{(R)}. \quad (5)$$

where

$$\frac{1}{\alpha}T_{\mu\nu}^{(R)} = \frac{1}{2}g_{\mu\nu}f(R) - R_{\mu\nu}f'(R) + (\nabla_\mu \nabla_\nu - g_{\mu\nu}\square)f''(R) + \mathcal{W}_{\mu\nu}, \quad (6)$$

and

$$\begin{aligned} \mathcal{W}_{\mu\nu} &\equiv \mathcal{W}_{\mu\nu}^{(2)} - \frac{1}{3}\mathcal{W}_{\mu\nu}^{(1)} \\ &= -\frac{1}{2}g_{\mu\nu}\square R - \square R_{\mu\nu} + \nabla_\rho \nabla_\mu R_\nu^\rho + \nabla_\rho \nabla_\nu R_\mu^\rho - 2R_\mu^\rho R_{\nu\rho} + \frac{1}{2}g_{\mu\nu}R_{\rho\lambda}R^{\rho\lambda} \\ &\quad - \frac{1}{3}(2\nabla_\mu \nabla_\nu R - 2g_{\mu\nu}\square R - 2RR_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R^2). \end{aligned} \quad (7)$$

Here $R_{\mu\nu}$ is the Ricci tensor, respectively. Also the prime is also indicative of a derivative with respect to R . Now if we consider the spatially flat FRW metric for the universe as

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1}^3 (dx^i)^2, \quad (8)$$

and $T_{\mu\nu}^{(R)} = g_{\mu\nu}T_\mu^{\nu(R)}$ then the set of field equations (5) reduce to the modified Friedmann equations in the framework of $f(R)$ -gravity as

$$3H^2 = \frac{\rho_R}{\alpha}, \quad (9)$$

$$-2\dot{H} - 3H^2 = \frac{p_R}{\alpha}. \quad (10)$$

The model can be considered as a standard model with the effect of the Weyl and $f(R)$ gravity modification contributed in the energy density and pressure of the Friedman equations. After some algebraic calculation, the field Eq. (6), when

$$R = 6\dot{H} + 12H^2, \quad (11)$$

corresponding to standard spatially-flat FRW universe for the 00 and ii components yields,

$$\frac{\rho_R}{\alpha} = -\frac{1}{2}f(R) + 3(\dot{H} + H^2)f'(R) - 3H\dot{R}f''(R) + \mathcal{W}_{00}, \quad (12)$$

$$\frac{p_R}{\alpha} = \frac{1}{2}f(R) - (\dot{H} + 3H^2)f'(R) + (\ddot{R} + 2H\dot{R})f''(R) + \dot{R}^2 f'''(R) + \frac{\mathcal{W}_{ii}}{a^2(t)}, \quad (13)$$

where

$$\mathcal{W}_{00} = 3 \left(\ddot{H}(1-H) - 4\ddot{H}H^2 + 2\dot{H}^2(1-2H-4H^2) - 8H^4 \right), \quad (14)$$

$$\frac{\mathcal{W}_{ii}}{a^2(t)} = 4\ddot{H}(6H^2 + H + 3\dot{H}) + \dot{H} \left(19H^2 - 12H + \frac{3}{2}\dot{H} + \frac{9}{2} \right) + \frac{3}{2}(3 + H^2). \quad (15)$$

Here $H = \frac{\dot{a}}{a}$ is the Hubble parameter and the dot denotes a derivative with respect to cosmic time t . Also ρ_R and p_R are the curvature contribution to the energy density and pressure.

The energy conservation laws are still given by

$$\dot{\rho}_R + 3H\rho_R(1 + \omega) = 0, \quad (16)$$

where $\omega = \frac{p_R}{\rho_R}$ is the equation of state (EoS) parameter due to the curvature contribution which defined as [48] and it's given by

$$\omega = -1 - \frac{6f'''\mathcal{A}^2 + 6f''(\mathcal{B} + 2H\mathcal{A}) - f'(\dot{H} + 3H^2) + \frac{1}{2}f + 4\ddot{H}\mathcal{C} + \dot{H}\mathcal{D} + \frac{9}{2} + \frac{3}{2}H^2}{18f''\mathcal{A}H - 3f'(\dot{H} + H^2) + \frac{1}{2}f + 3\mathcal{E} + 24H^2 - \frac{1}{12}\dot{H}^2}, \quad (17)$$

where

$$\begin{aligned} \mathcal{A} &\doteq \ddot{H} + 4H\dot{H}, \\ \mathcal{B} &\doteq \ddot{H} + 4H\ddot{H} + 4\dot{H}, \\ \mathcal{C} &\doteq 3\dot{H} + 6H^2 + H, \\ \mathcal{D} &\doteq 19H^2 - 12H + \frac{3}{2}\dot{H} + \frac{9}{2}, \\ \mathcal{E} &\doteq -(1-H)\ddot{H} + 4H^2(\ddot{H} + 2) + 8\dot{H}^2(H + \frac{1}{2})^2. \end{aligned} \quad (18)$$

In the case of $f(R) = 0$, from Eqs. (12) and (13) we have $\rho_R = 0$ and $p_R = 0$. Therefore Eqs. (9) and (10) transform to the usual Friedmann equations in GR. But for an arbitrary $f(R)$ as,

$$f(R) = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} + \frac{1}{12}\phi^2R, \quad (19)$$

in a FRW cosmological model, for only time dependent ϕ by invariance of the action under changing fields and vanishing variations at the boundary, the equations of motion for scalar fields ϕ become,

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{3}\phi R = 0. \quad (20)$$

and the equation of state (EoS) parameter is given by

$$\omega = -1 - \frac{(1-H)\ddot{H} + \frac{1}{3}\mathcal{F}\ddot{H} + \frac{1}{12}\mathcal{G}\dot{H} - \frac{5}{2}(5H^2-1)}{\mathcal{E} - \frac{1}{12}(\ddot{\phi} + \dot{\phi}^2)}, \quad (21)$$

where

$$\begin{aligned} \mathcal{F} &\doteq 4H(1+3H) + 12\dot{H}, \\ \mathcal{G} &\doteq 19H^2 - 12H - (48H^2 + 12H)\dot{H} + \frac{1}{6}\dot{\phi}^2 + \frac{9}{2}. \end{aligned} \quad (22)$$

Also the solution for $H(t)$, Eq. (9), provides a dynamical universe with contraction for $t < 0$, bouncing at $t = 0$ and then expansion for $t > 0$. The above analysis clearly can be seen in the numerical calculation given in Fig. 1.

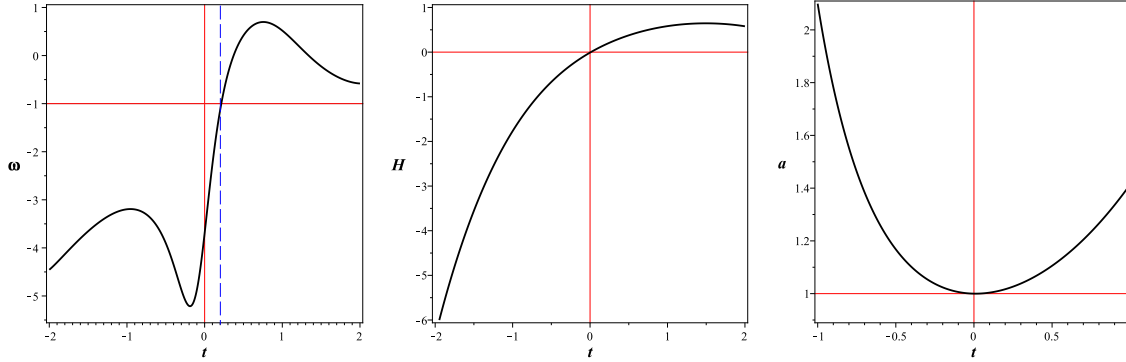


Fig.1: The graph of ω , H and scalar factor a , plotted as function of time.

Initial values are $\phi(0) = 0$, $\dot{\phi}(0) = 0.1$, $\dot{a}(0) = -0.01$ and $\alpha = 1$.

In our model, as [48], the EoS parameter crosses -1 line from $\omega < -1$ to $\omega > -1$, as Fig.1, which is supported by observations [16]. This model bears the same as quintom dark energy models which includes two quintessence and phantom fields [26]. For a successful bounce implying, a list of test on the necessary conditions is needed that during the contracting phase, the scale factor $a(t)$ should being decreased, i.e., $\dot{a} < 0$, and in the expanding phase, we should have $\dot{a} > 0$. At the bouncing point, $\dot{a} = 0$, and so around this point $\ddot{a} > 0$ for a period of time, the Hubble parameter H runs across zero from $H < 0$ to $H > 0$ and $H = 0$ at the bouncing point. Around bouncing point, for a successful bounce, the following condition should be satisfied

$$\dot{H} = -\frac{1}{\alpha}(1+\omega)\rho_R > 0. \quad (23)$$

According to Fig.1, at $t \rightarrow 0$, $\omega < -1$ and \dot{H} is positive and we see that at the bouncing point where the scale factor $a(t)$ is not zero, we avoid singularity faced in the usual Einstein cosmology.

At this stage, the cosmological evolution of EoS parameter, ω , is studied, and we show that, analytically and numerically, there are conditions that cause the EoS parameter cross the phantom divide line ($\omega \rightarrow -1$). Let us see under what conditions the system will be able to cross the barrier of $\omega = -1$. To do that, one needs $\rho_R + p_R$ to disappear at a point of (ϕ_0) and modify the sign after the crossing. One can accomplish this by requiring $H(\phi_0) = 0$ and H has different signs before and after the crossing. For investigating this possibility, we have to check the condition $\frac{d}{dt}(\rho_R + p_R) \neq 0$ when $\omega \rightarrow -1$. Using Eqs. (19) in (12) and (13) we have,

$$\frac{d}{dt}(\rho_R + p_R) = 3(1 - H)\ddot{H} + (\mathcal{F} - 3\dot{H})\ddot{H} + \mathcal{J}\ddot{H} - \mathcal{K}\dot{H}^2 + \mathcal{M} \neq 0, \quad (24)$$

where

$$\begin{aligned} \mathcal{J} &\doteq 12\ddot{H} + (12H + 19)\dot{H} + \mathcal{G}, \\ \mathcal{K} &\doteq 12 \left\{ 1 + (1 + 4H)\dot{H} \right\} - 38H, \\ \mathcal{M} &\doteq \frac{1}{3}\phi\dot{\phi} + 3H - 96H^3. \end{aligned} \quad (25)$$

In this case, our analytical discussion about $\omega \rightarrow -1$ would be just a boring game on different components of Eq.(24) to satisfy $\frac{d}{dt}(\rho_R + p_R) \neq 0$. For example if second and upper orders of derivatives of H respect to cosmic time t would have been vanished, one can find,

$$\dot{H}^2 \neq \frac{\phi\dot{\phi} + 3H(1 - 32H^2)}{6(12 - 19H)}. \quad (26)$$

3. $f(R)$ RECONSTRUCTION FROM HDE MODEL

There are two classes of scale factors which are ordinary investigated for explaining the accelerating universe in $f(R)$, $f(\mathcal{G})$ and $f(R, \mathcal{G})$ modified gravities. The first class of scale factor is donated by [49, 50]

$$a(t) = a_0(t_s - t)^{-h}, \quad t \leq t_s, \quad h > 0. \quad (27)$$

and consequently

$$H = \frac{h}{t_s - t} = \sqrt{\frac{h}{6(2h + 1)}}R, \quad \dot{H} = H^2/h, \quad (28)$$

which $\dot{H} > 0$ exhibits the model, matches with a phantom dominated universe. So, this model is usually so-called the phantom scale factor in the literature. For the second class [49]

$$a(t) = a_0 t^h, \quad h > 0, \quad (29)$$

with

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{h}{6(2h-1)}} R, \quad \dot{H} = -H^2/h, \quad (30)$$

which $\dot{H} < 0$ shows the model that describes a quintessence dominated universe. Therefore, this model is so-called the quintessence scale factor in the literature.

Here we reconstruct the Weyl gravity according to the HDE scenario. Following [51] the HDE density in a spatially flat universe is given by,

$$\rho_\Lambda = \frac{12\alpha c^2}{R_h^2}, \quad (31)$$

where $c = 0.818_{-0.097}^{+0.113}$ in recent observational data used to constrain the HDE model shows the flatten universe [52]. In addition R_h is the future event horizon defined as

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2}. \quad (32)$$

For the first class of scale factor, Eq.(27), with Eq.(28), the future event horizon R_h given by

$$R_h = \frac{1}{h+1} \sqrt{\frac{6h(2h+1)}{R}}. \quad (33)$$

Replacing Eq.(33) into Eq.(31) one can get

$$\rho_\Lambda = \frac{2\alpha c^2 (h+1)^2}{h(2h+1)} R. \quad (34)$$

Substituting Eq.(34) in the differential equation (12), i.e. $\rho_R = \rho_\Lambda$, gives the following solution

$$f(R) = \lambda_+ R^{m_+} + \lambda_- R^{m_-} + \gamma_1 R^{\frac{11}{2}} + \gamma_2 R^5 + \gamma_3 R^{\frac{3}{2}} + \gamma_4 R. \quad (35)$$

where

$$m_\pm = \frac{3+h \pm \sqrt{1-10h+h^2}}{4}, \quad (36)$$

and

$$\begin{aligned}
\gamma_1 &= 8\gamma_c h^2 \sqrt{\frac{6h}{2h+1}} \frac{(m_+ - m_-)}{(11 - 2m_+)(11 - 2m_-)}, \\
\gamma_2 &= \eta\gamma_c \frac{(m_+ - m_-)}{(5 - m_+)(5 - m_-)}, \\
\gamma_3 &= 12\gamma_c(2h+1) \sqrt{\frac{6h}{2h+1}} \frac{(m_+ - m_-)}{(3 - 2m_+)(3 - 2m_-)}, \\
\gamma_4 &= -18\gamma_c(2h+1) \frac{(m_+ - m_-)}{(1 - m_+)(1 - m_-)}, \\
\eta &= (6c^2 + 4)h^4 + (15c^2 + 4)h^3 + (12c^2 - 1)h^2 + 3c^2h, \\
\gamma_c &= \frac{-1}{3h^2\sqrt{1 - 10h + h^2}(2h+1)}.
\end{aligned} \tag{37}$$

Also λ_{\pm} are the integration constants that can be determined from the necessary boundary conditions. Following [53] the accelerating expansion in the present universe could be generated, if one consider that $f(R)$ could be a small constant at present universe, that is

$$f(R_0) = -2R_0 \quad \text{and} \quad f'(R_0) \sim 0. \tag{38}$$

where $R_0 \sim (10^{-33}\text{eV})^2$ is the current curvature. Applying the above boundary conditions to the solution Eq. (35) one can obtain

$$\lambda_+ = \frac{2R_0m_- - Q(R_0) + m_+P(R_0)}{m_+ - R_0m_-}, \tag{39}$$

$$\lambda_- = \frac{2R_0m_+ - Q(R_0) + m_-P(R_0)}{m_- - R_0m_+}, \tag{40}$$

where

$$P(R_0) = \gamma_1 R_0^{\frac{11}{2}} + \gamma_2 R_0^5 + \gamma_3 R_0^{\frac{3}{2}} + \gamma_4 R_0, \tag{41}$$

$$Q(R_0) = \frac{11}{2}\gamma_1 R_0^{\frac{9}{2}} + 5\gamma_2 R_0^4 + \frac{3}{2}\gamma_3 R_0^{\frac{1}{2}} + \gamma_4. \tag{42}$$

Replacing Eq.(35) into Eq.(12) and using Eq.(28) one can get the EoS parameter of the holographic $f(R)$ -gravity model as

$$\omega_R = -1 - \frac{2}{3} \frac{W(R)}{h} \tag{43}$$

where

$$W(R) = \frac{\frac{11}{2}\zeta R^{\frac{11}{2}} + 5\vartheta R^5 - \frac{3}{2}\theta^2 R^3 + \kappa R^{\frac{5}{2}} + \kappa'' R^2 + \xi' R^{\frac{3}{2}} + \sigma R + 2(m_+\varsigma_+ + m_-\varsigma_-) + \varepsilon}{\zeta R^{\frac{11}{2}} + \vartheta R^5 - \frac{1}{h}\theta^2 R^3 + \kappa' R^{\frac{5}{2}} + \kappa''' R^2 - \xi R^{\frac{3}{2}} + \sigma' R + \frac{2}{h}(\varsigma_+ + \varsigma_-)}, \tag{44}$$

and

$$\begin{aligned}
\zeta &= \gamma_1(7h - 90), & \vartheta &= 6\gamma_2(h - 12), & \theta &= \frac{2h}{3(2h + 1)}, & \kappa &= \sqrt{\frac{h}{6(2h + 1)}}, \\
\kappa' &= \frac{4(h - 1)}{h(2h + 1)}\kappa, & \kappa'' &= -\frac{38h^2 + 31h + 36}{12(2h + 1)}, & \kappa''' &= \frac{(h + 3)}{h^2}\theta, & \xi &= \gamma_3(h + 2), \\
\xi' &= (12h\kappa - \frac{3}{2}\xi), & \sigma &= \frac{1}{2}(45h - 4\gamma_4 - 9), & \sigma' &= -2h(\gamma_4 + 8), & \varepsilon &= -27(2h + 1)h, \\
\varsigma_{\pm} &= ((m_{\pm} - 2)h - 2m_{\pm}^2 + 3m_{\pm} - 1)\lambda_{\pm}R^{m_{\pm}}.
\end{aligned} \tag{45}$$

EoS parameter (50) corresponds to a phantom accelerating universe, i.e. $\omega_R < -1$ when $\frac{W(R)}{h} > 0$. Recent observational data indicates the EoS parameter ω_R at the present lies in a narrow strip around $\omega_R = -1$ and is quite consistent with being below this value. So, with $\frac{W(R)}{h} = 0$ and using boundary conditions (38) one can find $h = 0$ or $h = -\frac{1}{2}$ that is consistent with EoS parameter of cosmological constant.

For the second class of scale factor, Eq.(29), and using Eq.(30), the future event horizon R_h reduces to

$$R_h = \frac{1}{h - 1} \sqrt{\frac{6h(2h - 1)}{R}}, \quad h > 1. \tag{46}$$

where the condition $h > 1$ is obtained due to have a finite future event horizon. If we repeat the above calculations, the both of $f(R)$ and ω_R corresponding to the HDE for the second class of scale factor (29) will be yielded. Replacing Eq.(46) into Eq.(31) yields

$$\rho_{\Lambda} = \frac{2\alpha c^2(h - 1)^2}{h(2h - 1)}R. \tag{47}$$

The result for $f(R)$ is same as Eq.(35) where now

$$m_{\pm} = \frac{3 - h \pm \sqrt{1 + 10h + h^2}}{4}, \tag{48}$$

and

$$\begin{aligned}
\gamma_1 &= 8\gamma_c h^2 \sqrt{\frac{6h}{2h-1}} \frac{(m_+ - m_-)}{(11 - 2m_+)(11 - 2m_-)}, \\
\gamma_2 &= \eta\gamma_c \frac{(m_+ - m_-)}{(5 - m_+)(5 - m_-)}, \\
\gamma_3 &= -12\gamma_c(2h-1) \sqrt{\frac{6h}{2h-1}} \frac{(m_+ - m_-)}{(3 - 2m_+)(3 - 2m_-)}, \\
\gamma_4 &= 18\gamma_c(2h-1) \frac{(m_+ - m_-)}{(1 - m_+)(1 - m_-)}, \\
\eta &= (6c^2 + 4)h^4 - (15c^2 + 4)h^3 + (12c^2 - 1)h^2 - 3c^2h, \\
\gamma_c &= \frac{1}{3h^2\sqrt{1+10h+h^2}(2h-1)}.
\end{aligned} \tag{49}$$

Also the EoS parameter is obtained as

$$\omega_R = -1 + \frac{2}{3} \frac{W(R)}{h}, \tag{50}$$

but

$$\begin{aligned}
\zeta &= \gamma_1(7h + 90), & \vartheta &= 6\gamma_2(h + 12), & \theta &= \frac{2h}{3(2h-1)}, & \kappa &= \sqrt{\frac{h}{6(2h-1)}}, \\
\kappa' &= \frac{4(h-1)}{h(2h-1)}\kappa, & \kappa'' &= -\frac{38h^2 - 31h + 36}{12(2h-1)}, & \kappa''' &= \frac{(h-3)}{h^2}\theta, & \xi &= \gamma_3(h-2), \\
\xi' &= (12h\kappa - \frac{3}{2}\xi), & \sigma &= -\frac{1}{2}(45h + 4\gamma_4 + 9), & \sigma' &= -2h(\gamma_4 + 8), & \varepsilon &= 27(2h-1)h, \\
\varsigma_{\pm} &= ((m_{\pm} - 2)h + 2m_{\pm}^2 - 3m_{\pm} + 1)\lambda_{\pm}R^{m_{\pm}}.
\end{aligned} \tag{51}$$

which describes an accelerating universe with the quintessence EoS parameter when $0 < \frac{W(R)}{h} < 1$, i.e. $-1 < \omega_R < -\frac{1}{3}$

4. CONCLUDING REMARKS

In this paper, the evolution of the gravitational fields both analytically and numerically in the $f(R)$ modified gravity model coupled by the first gravitational theory was considered where it was invariant under the scale transformation and was presented by Weyl. A formulation of gravity as a simple modified model characterized by one scalar field ϕ which can be viewed in our example was taken into consideration as well. Analytical study of the solution indicates that under special condition, the universe may undergo a transition from quintessence to phantom phase which is also supported by numerical analysis. In analytic

studying of the dynamics of the EoS parameter we obtain the constraints that one has to impose on the scalar field and their first and second derivatives in order to have phantom crossing. In numerical approach, the EoS parameter crosses $\omega = -1$ for $t > 0$. We investigated about a bouncing non-singular cosmology, with an initial contracting phase which lasts until to a non-vanishing minimal radius is reached and then transits into an expanding phase which provides a possible solution to the singularity problem of Standard Big Bang cosmology, a problem which is not cured by scalar field driven inflationary models. The evolution of EoS parameter, hubble parameter and scale factor numerically obtained. The violations of the null energy condition required to get a bounce are obtained for the model, which allows a transition of the EoS parameter through the cosmological constant boundary. The result is that in the analytical discussion of the phantom crossing behavior of the EoS parameter, we have to also constrain the scalar field and their first and second derivatives. Besides, we have also additional constraints on hubble parameter and its first and second derivatives.

Furthermore, the HDE model which is begun from some important characteristics of quantum gravity and is motivated from the holographic hypothesis, was investigated in the framework of $f(R)$ -gravity. A natural unification of the early-time inflation and late-time acceleration because of different role of gravitational terms relevant at small and at large curvature and may naturally describe the transition from deceleration to acceleration in the cosmological dynamics was given by modified gravity. The modified gravity based on the $f(R)$, coupled by Weyl tensor, action in the spatially flat FRW universe for the two class of scale factor, according to the original version of the HDE scenario was reconstructed and the EoS parameters of the corresponding our model was given. Our considerations indicated, for the first class of scale factor, the EoS parameter always crosses the phantom-divide line, whereas for the second class, behaves like the quintessence. Furthermore our model corresponding to the HDE can predict the early-time inflation of the universe.

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